CHAPTER - 1

REAL NUMBERS

KEY CONCEPTS AND FORMULAE

- THE FUNDAMENTAL THEOREM OF ARITHMETIC: Every composite number can be expressed as a product of primes, and this factorization is unique except for the order in which the prime factors occur.
- Every composite number can be uniquely expressed as the product of powers of primes in ascending or descending order.
- For any two positive integers a and b, HCF (a, b) x LCM(a, b) = a x b.
- Let 'a' be a positive prime number such that 'p' is divisible by a^2, then p is also divisible by a.
- There are infinitely many positive primes.
- If p is a positive prime, then $\sqrt{p}$ is an irrational number. For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$ etc. are irrational numbers.

SOLVED EXAMPLES

EXAMPLE 1: Show that 12^n cannot end with digit 0 or 5 for any natural number n.

SOLUTION: Expressing 12 as the product of primes, we obtain

$12 = 2^2 \times 3$,

$12^n = (2^2 \times 3)^n = (2^2)^n \times 3^n = (2)^{2n} \times 3^n$

So, only primes in the factorization of 12^n are 2 and 3 and, not 5. Hence, 12 cannot end with digit 0 or 5.

EXAMPLE 2: Find the HCF of 96 and 404 by prime factorization method. Hence, find their LCM.

SOLUTION: We have, $96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

Thus, HCF = $2^2 = 4$

Now, HCF x LCM = Product of numbers

= 96 x 404,

LCM = (96 x 404)/HCF = (96 x 404)/4 = 96 x 101 = 9696

EXAMPLE 3: On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance and complete steps?

SOLUTION: Each person will cover the same distance in complete steps if the distance covered in cm is the LCM of 40, 42 and 45.

Now, $40 = 2^3 \times 5$, $42 = 2 \times 3 \times 7$ and $45 = 3^2 \times 5$

LCM of 40, 42 and 45 is $2^3 \times 3^2 \times 5 \times 7 = 2520$

Hence, minimum distance each should walk = 2520 cm.

EXAMPLE 4: Prove that $5 - \sqrt{3}$ is an irrational number.

SOLUTION: Let us assume on the contrary that $5 - \sqrt{3}$ is rational. Then, there exist co-prime positive integers a and b such that

$$5 - \sqrt{3} = a/b$$

$$\Rightarrow 5 - a/b = \sqrt{3}$$

$$\Rightarrow (5b - a)/b = \sqrt{3}$$

Since, a and b are integers and thus $(5b - a)/b$ is rational number. Thus $\sqrt{3}$ is rational but this contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect. Hence, $5 - \sqrt{3}$ is an irrational number.

EXAMPLE 5: Prove that $3 + 2\sqrt{5}$ is irrational.

SOLUTION: Let us assume on the contrary that $3 + 2\sqrt{5}$ is rational. Then, there exist co-prime positive integers a and b such that

$$3 + 2\sqrt{5} = a/b$$

$$\Rightarrow 2\sqrt{5} = a/b - 3$$

$$\Rightarrow \sqrt{5} = (a - 3b)/2b$$

Since, a and b are integers and thus $(a - 3b)/2b$ is rational number. Thus $\sqrt{5}$ is rational but this contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect. Hence, $3 + 2\sqrt{5}$ is an irrational number.

PRACTICE QUESTIONS

MULTIPLE CHOICE (1 mark)

1. If p and q are two distinct prime numbers, then HCF is

   a) 2   b) 0   c) either 1 or 2   d) 1

2. If p and q are two distinct prime numbers, then LCM (p, q) is

   a) 1   b) p   c) q   d) pq

3. Let p be a prime number. The sum of its factors is

   a) p   b) 1   c) p + 1   d) p – 1

4. The LCM of the smallest two digits composite number and the smallest composite number is

   a) 12   b) 20   c) 4   d) 44

5. The HCF of smallest prime number and the smallest composite number is

   a) 2   b) 4   c) 6   d) 8

6. The smallest number divisible by all the natural numbers between 1 and 10 (both inclusive) is

   a) 2020   b) 2520   c) 1010   d) 5040

7. Let n be a natural number. Then, the LCM (n, n+1) is

   a) n   b) n + 1   c) n (n + 1)   d) 1

8. If 3 is the least prime factor of m and 5 is the least prime factor of n, then the least prime factor of (m + n) is

   a) 11   b) 2   c) 3   d) 5

9. If HCF (x, 8) = 4, LCM (x, 8) = 24, then x is

   a) 8   b) 10   c) 12   d) 14

10. If two positive integers m and n are expressible in the form $m = pq^3$ and $n = p^3q^2$, where p, q are prime numbers, then HCF (m, n) =

    a) pq   b) pq^2 c) p^3q^3 d) p^2q^3

11. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then n =

    a) 1   b) 2   c) 3   d) 4

12. If n is any natural number, then $6^n - 5^n$ always end with

    a) 1   b) 3   c) 5   d) 7

ASSERTION AND REASONING (1 mark)

Each of the following examples contains Assertion (A) and Reason (R) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) A is true, R is true; and R is correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true, R is false.
(d) A is false, R is true.

1. A: If LCM (60, 72) = 360, then HCF (60, 72) = 12.
   R: HCF (a, b) x LCM (a, b) = a + b.
2. A: The product of (5 + √3) and (2 - √3) is an irrational number.
   R: The product of two irrational numbers is an irrational number.
3. A: HCF and LCM of two natural numbers are 25 and 815 respectively.
   R: LCM of two natural numbers is always divisible by their HCF.
4. A: HCF (234, 47) = 1.
   R: HCF of two co-primes is always 1.
5. A: √11 is an irrational number.
   R: If p is a prime number, then √p is an irrational number.

VERY SHORT ANSWER TYPE (2 marks)

1. **Given that HCF (2520, 6600) = 40 and LCM (2520, 6600) = 252 x k, then find the value of 'k'.
2. *If two positive integers a and b are written as $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers, such that HCF (a, b) = p^mq^n and LCM (a, b) = p^rq^s, then find the value of (m + n)(r + s).
3. *Three bells ring at intervals of 4, 7 and 14 minutes. All the three rang at 6 AM. When will they ring together again?
4. *Determine the prime factorization of 58500.
5. **Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 are composite numbers.
6. *Check whether 6^n can end with the digit 0 for any natural number n.
7. *The HCF of two numbers is 16 and their product is 3072. Find their LCM.
8. Write the smallest number which is divisible by both 306 and 657.

SHORT ANSWER TYPE (3 marks)

1. **Find the HCF and LCM of 144, 180 and 192 by prime factorization method.
2. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.
3. In a school there are two sections – section A and section B of class X. There are 32 students in section A and 36 students in section B. Determine the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.
4. **Find the LCM and HCF of the pair of integer 404 and 96 and also verify that LCM x HCF = Product of the integers.
5. *On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
6. *Prove that $\sqrt{2} + \sqrt{5}$ is irrational.
7. *Show that $3 + \sqrt{2}$ is an irrational.
8. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then find the product of two numbers.

LONG ANSWER TYPE (5 marks)

1. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively.
2. ***Prove that $\sqrt{2}$ is an irrational number.
3. **Prove that $\sqrt{3}$ is an irrational number.
4. **Prove that $(2 + \sqrt{3})/5$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

CASE STUDY BASED (4 marks)

1. **Mira is very health conscious and avoids fast food, cakes, ice-creams etc. On her birthday she decided to serve fruits to her friend guests. She had 60 bananas and 36 apples which are to be distributed equally among all.**

(i) How many maximum guests Mira can invite? (1 M)

(ii) How many apples will each guest get? (1 M)

(iii) If Mira also decide to distribute 42 mangoes, how many maximum guests she can invite. Also, find the total number of fruits that each guest will get. (2 M)

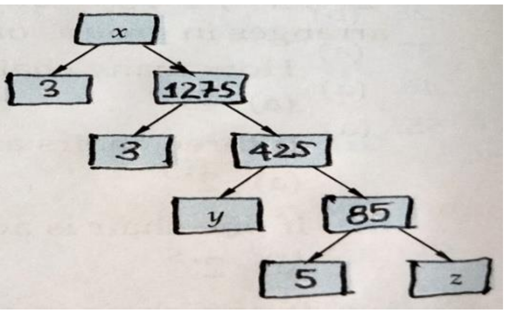
2. **A seminar is being conducted by an educational organization, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.**

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence find the maximum number of participants that can be accommodated in each room. (1 M)

(ii) Find the minimum number of rooms required during the event. (1 M)

(iii) Find the product of HCF and LCM of 60, 84 and 108. (2 M)

3. **Observe the factor tree below and answer the questions:



(i) Find the value of y. (1 M)

(ii) Find the value of z. (1 M)

(iii) Determine the value of x + y + z. (2 M)

ANSWER KEY CHAPTER-1

MULTIPLE CHOICE:

1. d 2. d 3. c 4. b 5. a 6. b 7. c 8. b
9. c 10. b 11. b 12. a

ASSERTION AND REASONING:

1. c 2. c 3. d 4. a 5. a

VERY SHORT ANSWER TYPE:

1. 1650 2. 35 3. 6:28 AM 4. $2^2 \times 3^2 \times 5^3 \times 13$

5. Since 7 x 11 x 13 + 13 =13 (7 x 11 + 1) and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5(7 x 6 x 4 x 3 x 2 x 1 + 1)

6. $6^n = (2 \times 3)^n = 2^n \times 3^n$ 7. 192 8. 22338

SHORT ANSWER TYPE:

1. HCF = 12 ; LCM = 2880 2. 17 3. 288 Books 4. LCM = 9696 ; HCF = 4 5. 360 cm 8. 194400

LONG ANSWER TYPE:

1. 625

CASE STUDY BASED:

1. (i) 12 (ii) 3 (iii) 6 ; 23
2. (i) 12 (ii) 5 (iii) 45360
3. (i) 5 (ii) 17 (iii) 3847